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QUOTATIONS
ANCIENT INDIAN SCIENCE

(MATHEMATICS & ASTRONOMY)

Dr. N. GOPALAKRISHNAN

INDIAN INSTITUTE OF SCIENTIFIC HERITAGE
THIRUVANANTHAPURAM

Heritage Publication Series - 3

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QUOTATIONS: ANCIENT INDIAN SCIENCE

(Third Edition)

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INDIAN INSTITUTE OF SCIENTIFIC HERITAGE, THIRUVANANTHAPURAM

World over renewed interests are observed on ancient Indian Scientific and spiritual knowledge. Series of research activities are undertaken by universities and research institutions in many countries. True search of good, adoptable and useful scientific observations on Indian approaches on all aspects of life are being conducted by scholars. Results of these studies are astounding. Yoga has found a respectable place in the medical curriculum and medical books published even from America. Quantum holistic healing is finding it's benefit for many diseases. Vedic mathematics is getting into the school and college syllabus in many countries of Europe and UK. Medicinal plants and Indian alternatives for eco-friendly pesticides are going into large scale production and utilisation in developed countries. Vegetarianism is becoming a precious life style. Meditation goes into stress relief in homes and production centres. The crude tribal medicines go for curing even complicated health problems. The massaged oil bath has become an attraction for west for a comfortable physique. Sanskrit manuscripts find a respectable place even in the most modern education centres, libraries and archives. Effect of mantras are being studied under a new brand name 'psycholinguistic effects'. Curative property of turmeric and pesticidal activity of neem are hot subjects in patenting, and so on many are in the list...... To conclude the advantages and usefulness of ancient Indian knowledge, it may be better to say that even the Panchatantra stories, our children's favourite story book, has been selected as a course book for training the American intelligence/ investigation agency.

The above adoptions occurred as a result of systematic and in-depth studies of the Indian Scientific values by national and international scholars. It is important to search facts on Indian inventions and discoveries which were neither appreciated nor recognised. Many examples can be cited; Hippocrates, is considered as the father of modern medicine, even though he collected almost all information from Charaka and Susrutha Samhitas! Baudhayana propounded the theorem four hundred years before Pythagorus, but

Baudhayana was forgotten! Bhaskaracharya II gave specific details on gravity, three hundred years before Newton. Why Bhaskaracharya was not recognised? Sangama grama Madhava described the mathematical series earlier than Gregory, still the series is known as Gregory's series! Charaka described in five passages the blood circulation in 300 BC, but William Harvey got the garland! Aryabhatta gave in detail with exact values the explanations on the spherical nature, diameter, speed of rotations and number of rotation of earth, as given in modern science, in AD 499, but the credit was given to Kepler, Galileo and Copernicus of 15th century. Brahmagupta's equation for area of quadrilateral has become Style's equation! Puthumana Somayajee's equation is known as De Moivere's equation! Madhava's equation is known as Gregory's equation! Neelakanta's equation is known as Newton's equation! Sayana's name is never mentioned when velocity of light is discussed! Even, Apianus, Stiefel, Scheubel, Tartaglia and Bombelli'borrowed' almost all from Indian books and they became discoverers and inventors, whereas the real Indian Scientists were ignored. Many more examples can be cited! Every ancient Indian contribution had the fate of Baudhayana's theorem converting to Pythagorus theorem! The only reason for this neglect may be that those scientists were Indians. However, it is to be brought to the attention of the world that Indian contributions should be recognised as INDIAN! All scientifically meritorious facts from India should be taught and preached for practising, to begin with at least in India! Unfortunately, now there are very few forums in India, to bring together the students and scholars interested in this 'new old' area of study. **Indian Institute of Scientific Heritage (IISH, Thiruvananthapuram)** is a humble venture of a group of scientists of different subjects for the purpose. Through publications, recorded cassettes, poster exhibitions, seminars and study classes, **IISH** would like to fulfil the noble-divine-scientific-patriotic mission. Your presence, encouragement, guidance and blessings are inevitable for the mission. This book, on the quotations of ancient Indian sciences is submitting to you!.

Dr. M. Sambasivan, FRCS
Chairman

Dr. N. Gopalakrishnan, PhD
Hon. Director

MODERN CONCEPTS IN ANCIENT INDIAN SCIENCES

There are different views on ancient Indian scientific heritage. Few are of the view that all the modern basic and technical knowledge were available in ancient India. Second view is that nothing worth mentioning science existed in India. The latter say that what we had, was only some philosophy and spirituality. The third group has the opinion that some science, like pythagorus theorem, value and use of zero, etc were here. They also quote puranas (better to say misquote) to say that Agneya Astra may be atom bomb, Dronacharya is a test tube baby and so on. From an unbiased approach based on the modern scientific temper in it's true sense, one can say that all the above views are equally wrong.

Without touching the Vedas and puranas or even Itihasas, one can directly search out the scientific information on almost all fields of science, in specific ancient Indian scientific books, written in Sanskrit. Hundreds of books are available in original Sanskrit with English/ Hindi translations. Majority of these books are the sources of verifiable knowledge in mathematics, astronomy, metallurgy, health science, agricultural science, and so on. First hand knowledge on the technical capability of ancient Indians can also be observed from the building structures, metallic and ceramics materials available from archeological evidences. Archeological study results are available from 5000 BC onwards, perhaps further back!

A series of mathematics and astronomical books written in Sanskrit are available in the printed form. Unfortunately common man thinks that these are part of only astrology. No! All these information are present in astronomical books. (In Sanskrit the word Jyothisastra is used for both astronomy and astrology). The quotation (in the English translation form) are taken from these astronomy and mathematics books. They do not contain anything on the

prediction part of astrology. Many of these knowledge are well defined theories and observations in modern science. Unfortunately almost all these information are taught even in India as the contributions of Western Scientists. Many of the theorems cited here are known under the names of Newton, Galileo, Kepler, Euler, Style, Haron, and so on. The fact is that these theorems were known to our forefathers, even a millennia before, the modern scientists rediscovered them.

Not only mathematical and astronomical subjects were dealt in detail in this coutnry, but also subjects like, ceramics, metallurgy, soil chemistry, glass, agriculture, plant drugs, drug processing, surgery, architecture, food science, modern physical observations, metal finishing, alloy preparations, textile industry, sericulture, fermentation, etc..... were not confined to imaginary theoretical knowledge, but were practiced, in nook and corner of this land.

Ancient Indian scientific heritage is the source of inspiration. It is an important means to project our nation. It is an important source of new research material, product, technology and ideas. It is also a matter of giving factual information on our science and scientists to the younger generation of the world, particularly to Indians. The most important use of the traditional knowledge is it's utilisation as a source of new idea for modern research. Making modern life more pleasant and happy!

Period of the ancient Indian Books quoted here is separately given, at the end!

1-10-1999 **Dr. N. Gopalakrishnan**

ANCIENT INDIAN SCIENTIFIC TERMS AND THEIR EQUIVALENTS IN MODERN SCIENCE

Akshajya	: Rsine of latitude	- A
akshakarna	: equinoctial midday shadow	- B
akshakoti	: colatitude	- B
akshachapa	: arc of latitude	- B
akshamsa	: latitude of the place	- B
akshonnati	: inclination of the earth's axis	- B
asita	: measure of moon's unilluminated part	- V
astarka	: sun's position at the time of star's heliacal setting	- V
agra	: amplitude of the rising or Rsine of that	- A
agra	: the arc of celestial horizon lying between the east point and the point where a heavenly body rises, or between the west point and the point where a heavenly body sets	- B
apakrama	: greatest declination	- A
apamandala	: ecliptic	- A
adhva	: distance of place from meridian	- B
ardhajya	: Rsine	- A
astamayodayasutra	: rising and setting line	- A
ativakra	: midst of retrograde motion	- B
ahoratrardhavishkambha	: day radius	- A
avanama	: zenith distance	- B
astalagna	: setting pont of the ecliptic	- B
ayanachalana	: Precision of the equinoxes	- B
udayajya	: the rising point of planet's orbit	- B
utkramana	: R verse of sine	- A
uttarayana	: sun's northward journey from winter solistics to summer solistics	- A

unnatabhaga	: degrees of latitude	- A
unmandala	: equitorial horizon	- A
kakshyamandala	: mean orbit of planet	- A
kalardhajya	: Rsine difference in terms of angle minutes	- A
kuavarta	: rotation of earth	- A
kshitija	: horizon	- A
kshitijya	: earthsine. The distance between the rising setting line and the line joining the point of the intersection of the diurnal circle and six o' clock circle	- B
kshitijaa	: earthshine	- A
kshepa	: celestial latitude	- A
konamandala	: intermediary vertical circle	- L
krantimandala	: ecliptic	- L
ghanabhoomadhya	: centre of earth	- A
chandroccha	: apogee of moon	- A
chara	: Ascensional difference, it is defined by the arc of the celestial equator lying between the six o' clock circle and hour circle of a heavenly body at rising	- B
chayabhramana	: the path of the last point of the shadow	- B
jeevabhukti	: true daily motion derived mathematically	- B
tamovishkambha	: diameter of shadow	- A
taragraha	: real planets	- A
akshavala	: deflection due to latitude	- A
dyudalasanku	: Rsine altitude at midday	- A
drikshepa	: ecliptic zenith distance	- A
drukshepa	: shortest arcual distance of the planet's orbit from the zenith. It is also used for the Rsine of distance	- B

desantara	: the longitude of the place. It is also the distance of local place from the prime meridian or the difference between the local and the standard times	- B
desantra karma	: correction of the longitude for the place	- B
drkshepamandala	: vertical circle through central ecliptic point	- A
drkchaya	: parallax	- A
drggati	: arc of ecliptic between sun or moon and central ecliptic point	- A
drggola	visible celestial sphere	- A
drngmandala	: visible vertical sphere	- A
dhyujyamandala	: diurnal circle	- L
natajya	Rsine of the zenith distance	- A
nakshatra /divas	: sidereal day	- A
neecha	: perigee	- A
palabha	: equinoctial shadow	- L
purvaparasutra	: east-west line	- L
paramapakrama	: greatest declination, obliquity of ecliptic	- A
paramasanku	: Rsine of greatest altitude/ meridian altitude	- A
parilekha	: graphical representation	- L
pratiloma	: retrogression	- A
praglagna	: rising point of ecliptic	- A
pragjyakashta	: right ascension	- A
bhabhrama	: locus of shadow	- V
bhakakshya	orbit of asterisms	- V
bhavrutta	: shadow circle	- V
bhoogola	: sphere of earth	- A
bhoodivas	: terrestrial day	- A
bhramavrutta	: diurnal circle or day circle	- V

■

madhygraha	: mean planet	- A
madhyjya	: Rsine of the zenith distance of the meridian ecliptic point	- A
mandakendra	: longitude of the planet minus longitude of the planet's apogee	- B
madhyamakranti	: declination of place of planet on ecliptic	- L
mandagatiphala	: correction to mean motion of planet	- L
nati	: meridian zenith distance. Celestial latitude as corrected for the parallax in latitude	- B
natakala	- hour angle	- L
natakalakotijya	: Rcosine of the hour angle	- L
natakalajya	Rsine of the hour angle	- L
nadivalaya	: celestial equator	- L
neechochharekha	- line of apses	- L
valana	. defelction relates to an eclipsed body It is the angle subtended at the body by the arc joining the north pole of the ecliptic. The akshvalana is the angle subtended at the body by the arc joining the north point of the celestial horizon and the north pole of the equator. Ayanavalana angle subtended at the body by the arc joining north pole of equator and ecliptic	- B
vikshepa	: celestial latitude	- A
vishuvat	: equator	- A
samamandala	: prime vertical	- L
sankvarga	. distance of planet's projection on plane of horizon	
from	: rising setting line	- A

suddhi	mean heliocentric position	- L
sphutakranti	declination of planet's centre	- L
sphutavikshepa	celestial latitude corrected by parallax	- L
sphutamadhya	true mean planet	- A
sphuta	. true planet	- A

A : Aryabhateeya of Aryabhatta, B : Mahabhaskareeya of Bhaskaracharya I, L : Sishyadhi vruddhi Tantra of Lallacharya, V : Vateswara Siddhanta of Vateswara.

ASTRONOMY

PLANETS

Mathematician who is expert in spherics, knows the path of planets, i.e astronomy If one knows other than this, how can he understand the movement of the planets. (Jishnu Nandana)

Know that all planets are spherical. And also know that it revolves in the eccentric (pratimandalam) orbits which are equal to the circular orbit of the planets (Siddhanta darpana - Neelakanta)

In the group of great celestial bodies, all the planets made of panchabhutas, exist in space, like a magnet attract a piece of iron, from all the sides. (Varahamihira-Pancha siddhantika)

Due to the powerful light from the sun, the planets also appear as shining, in the space. Due to this light, we see the planets (Neelakanta-Aryabhateeya golapada)

Rising of the planets

Ascending nodes of Mercury, Venus, Mars, Jupiter and Saturn are 20°, 60°, 40°, 80°, and 100° from the first point of the Sign Aries (Aryabhateeyam 1-9a)

When the sighrakendras (apogees) of Mars, Mercury, Jupiter, Venus and Saturn are respectively 28°, 205°, 14°, 183° and 20° they

rise in the east. When they are 360° minus these values respectively they set in the east. Mercury and Venus once set, rise again 32 and 71 days respectively after setting. For 120, 16, 30, 8 and 36 days respectively, Mars, etc., (in that order as given above) remain invisible. They set 660, 37, 372, 251 and 342 days respectively after rising (Sishyadhi Vruddhi Tantra 3-22-25)

The planets which have a motion slower than the Sun always rise helically in the east and sets in the west. The planet which has a faster motion, rise in the west and set in the east. So, Mars, Jupiter and Saturn which have slower motion than the sun always rise in the east and set in the west. Mercury and Venus when retrograde do the same, But when they have direct motion - which is faster than Sun- they rise helically in the west and set in the east. (Sishyadhi Vruddhi Tantra 8-1)

On Saturday noon, the beginning of Caitra, Saka year 854. The position of Sun is 11s16°12′, Moon 11s 18°22′, Apogee of sun 2s18° or 78° apogee of moon or 0s7° 20′ that of Mars 4s7°, Mercury, Jupiter, 5s 22°, Venus 2s20°, Saturn 247°. (Laghumanasa of Manjulacharya)

At the end of completion of Sun's revolution in Saka year 854, the ascending nodes are for Moon 8s9°56′, Mars 40°, Mercury 20°, Jupiter 80°, Venus 60°, Saturn 100°. Ayana chalana (precision of equinoxes) 6°50′ and rate of ayanachalana is 1′ per year (Laghumanasa)

Declinations of the planets are in this order. The greatest declination of sun is 24°, for moon 4½°, Saturn, Jupiter, Mars have respectively 2°,1°, and 1½° for Mercury and Venus, the declinations are 2° each. 96 angula, or 4 cubit make a Nr unit of measurement (Aryabhateeyam 1-8)

One half of the ecliptic running from the beginning of the sign Aries to the end of the sign Virgo, lies obliquely inclined to the equator northwards. The remaining half of the ecliptic running from the beginning of the sign Libra to the end of the sign Pisces lies equally inclined to the equator to southwards (Aryabhateeyam 4-1)

Visibility of the planets

Planet appear small or accordingly, in the same manner, they appear small or large according as they are near the sun or far from it (Sishyadhi Vruddhi Tantra 14-11)

It is because the observer is standing on the surface of earth and also because of the parallax, the observer sees the visible and non visible part of the planet as big and or small (Aryabhateeyam)

When the moon has no latitude it is visible when situated at a distance of 12 degree equivalent of time from the sun. Venus is visible when 9° of time distance from the sun. The other planets taken in the order of decreasing size Jupiter, Mercury, Saturn, and Mars are visible when they are 9°, increased by 2° (i.e11°, 13°, 15° and 17°) distant from sun (Aryabhateeyam 4-4)

Corrected by the two visibility corrections, the Moon, Mars, Jupiter and Saturn are helically visible when separated from the sun by (a degree or time equivalent to that) which are respectively 12°, 17°, 11° and15° Mercury and Venus are helically visible when in direct motion separated from Sun by a kalamsa 13° and 9° when retrograde by kalamsa 12° and 8° respectively. (Sishyadhi Vruddhi Tantra 8-5)

Visibility correction of planets

Multiply the Rversed sine of the true longitude of a planet increased by 90° by its latitude and by 60 and 50/60 and divide the product by 22500. The result in minutes is the dark phase or visibility correction of the planet due to the obliquity of the ecliptic (Sishyadhi Vruddhi Tantra 13-14)

OR multiply the difference of the radius and the R sine of the true longitude of planet by its latitude and divide by 370. The result is the visibility correction (in minutes of angle) (Sishyadhi Vruddhi Tantra 13-15).

Parallax on visibility

Because of the spherical shape of the earth, the parallax equal to

a maximum level of earth's radius in angles is possible for observing the celestial bodies, from the earth (Aryabhateeyam 4-34)

R sine of the hour angle at the amavasya (new moon day) multiplied by the R sine of the altitude of the meridian ecliptic point and divided by 2954961 gives the parallax in ghatikas at mid eclipse (Sishyadhi Vruddhi Tantra 6-8)

The driggatijya divided by 860 gives the parallax in ghatikas. It should be added to the calculated time when the amavasy ends, if the sun is in the western hemisphere. The longitude of sun and moon must be found for the corrected time by adding or subtracting the minutes resulting (from the motion according as the parallax is additive or subtractive) (Sishyadhi Vruddhi Tantra)

The radius and the valanajya when divided by 5, are converted into angulas. The R sine of the driggati multiplied by 2 and divided by 75 gives the ghatika of the parallax in longitude. (Sishyadhi Vruddhi Tantra 13-11)

Rsine of the hour angle multiplied by the Rsine of the altitude of the meridian ecliptic point and divided by 5625 gives parallax in longitude. The difference of the true motions of the Sun and Moon multiplied by the Rsine of the drikshepa and divided by 2250 gives the parallax in latitude (Sishyadhi Vruddhi Tantra 13-12)

Apogees of the planets

When a planet is at a distance of 6 signs (180°) from its apogee, it is said to be at the perigee or nicha. When a planet is at the apogee, it is farthest from the earth when at the perigee, it is nearest to the earth. This is so because of the length of the hypotenuse in each case (Sishyadhi Vruddhi Tantra 14-10)

Apogees of the Mars, Mercury, Jupiter, Venus and Saturn are respectively 118°, 210°, 180°, 90° and 236° The circumferences of their manda cycles divided by 4½. Or mandagunakas are 14°, 7°, 7°, 4°, 9° respectively. The circumference of their sigra epicycles are 53°, 31°, 16°, 59°, and 90° (Sishyadhi Vruddhi Tantra 3-1)

When compared to the apogees of the star planets, for the Sun it is highest, for the planets in the order of Mars, etc the degree of arc of the epicycle forming the part of the apogee is 35, 14, 16, 2, and 24 are multiplied with 2 in that order (Pancha Siddhantika 17-1)

The manda epicycles of the Moon, Sun, Mercury, Venus, Mars Jupiter and Saturn in the first and third anomalistic quadrants are respectively 7,3,7,4,14,7 and 9 each multiplied by 4½. The sigra epicycles of Saturn Jupiter Mars Venus and Mercury are 9, 16, 53, 59 and 31 degrees each multiplied by 4½ (Aryabhateeyam 1-10)

The manda epicycles of the retrograding planets in the same order from Mercury are, in the second and fourth anomalistic quadrants, 5 2,18, 8 and 13 each multiplied by 4½. And the sighra epicycles in the reverse order of the planet are 8,15 51,57 and 29 multiplied by 4½ (Aryabhateeyam 1-11)

The first quadrant extends from 0°-90° plus 45°, 31°, 20°, 50°, and 6° for Mars, Mercury, Jupiter Venus and Saturn respectively. The third quadrant/ pada extends from 180° to 270° minus 45°, 31°, 20°, 50° and 6° respectively for these planets. This padas should be kept in view while applying the sigra gatiphala positively or negatively to the motion (Sishyadhi Vruddhi Tantra 11-14)

Apogees of the Sun, Mercury, Venus, Mars, Jupiter and Saturn are respectively 78°, 210°, 90°, 118°, 180°, 236° (Aryabhateeyam 1-9)

For finding the apogee of Mercury, multiply the ahargana with 100 and divide by 8797 add to the non fraction part of the quotient 4½" (Pancha siddhantika 16-7)

Multiply 21600 by 10 and then the revolution of the Moon in a Yuga. The result is the yojana in the circumference of the sky upto which the rays of the Sun reach (Sishyadhi Vruddhi Tantra 5-3)

The ecliptic intersects the celestial equator at the first point of Aries and Libra. It is the north of the equator by 24° where is the first point of cancer It is also to the south of equator by 24° where is the first point of Capricorn (Sishyadhi Vruddhi Tantra 15-7)

The revolutions of the celestial bodies and planets are never equal everywhere, when viewed from earth. That knowledge is explained

in astronomy known as spherical bodies (golam) (Neelakandta, Golapada)

In one Yuga, the total number of years is 43 lacs and 20 thousand (Mallikarjuna Suri 3-5)

The number of revolutions of Moon in a Yuga is 57753336 that of Mars is 2296824, and of earth is 1582237500, The revolution of sighrocca of Mercury is 17937020 that of Jupiter is 364224, sighrocca of Venus is 7022388 that of Saturn is 146564 (Mallikarjuna Suri 4-6)

In a Yuga the eastward revolution of the Sun is 4320000 of the Moon 57753336 of Saturn 146564, Jupiter 364224, Mars 2296824 of Mercury and Venus the same as that of Sun (Aryabhateeyam 1-3)

Revolutions of Sun 4320000, that of Moon 57753335, Mars 2296824, Jupiter 364224, Saturn 146564 that of Venus and Mercury equal to that of Sun (Laghubhaskareeyam 1-9-11)

Motion of Planets

Clockwise, anticlockwise, rotation, slow, extra slow, mean, fast, extra fast are the 8 types of motions of planets (Suryasiddhanta 2-12)

In the process of rotation of planets in a circular path, from a manda speed, the planet moves clockwise and from sighra it moves anticlockwise in the circular pathway (Golapada-Nilakanta)

The orbit which is eccentric/elliptic is known as elliptical orbit, movement through that orbit is the elliptical movement (Nilakanta)

The Velocity of true planet moving on the sigra epicycle is same as velocity of the true mean planet moving in its orbit (Aryabhateeyam 3-25b)

Reduce moons revolutions in a yuga to sign, multiplying with 12, this when multiplies respectively with 30,60 and 10 gives the total degrees, minutes and yojanas through which the moon had moved. (Aryabhateeyam 1-6) Similarly the orbit, i.e the distance travelled by each planet can be found out by multiplying the total revolution with 12, 30, 60 and 10 respectively.

The daily mean motion of the sun, etc., (Moon, Mars, apogee / sighrocca of Mercury, Jupiter, sighrocca of Venus, Saturn, and Moon apogee and node are respectively 59′8″, 790′35″, 31′26″, 245′26″, 5′0″, 96′8″, 2′0″, 6′41″, and 3′11″ (Sishyadhi Vruddhi Tantra 1-40, 41)

Multiply the degree of the Moon's true daily motion after reducing 11 from it, with R cosine of the true longitude of Sun minus the longitude of the moon's apogee. This is the multiplier of the Rsine and Rcosine of the true longitude of the Moon diminished by that of the Sun, divided by 1 and 5 respectively. This will give the correction, in terms of minutes of arc, for the moon and if the factor is positive and the other negative, the correction for the Moon is subtractive and that for its true daily motion additive. If both are of like signs, (both positive or both negative) the corrections are to be applied contrarily (Laghumanasa 4-1,2) i.e 8°8′cos(S-U) (Moon's true daily motion-11) 8°8′ sin (M-S). Where S, M, U denote the true longitudes of the Sun, Moon and the Moon's apogee (Laghumanasa 4-1,2)

Subtract 2 times the years elapsed from the Dvguna and divide that by 9. To this add 40 times the years elapsed. These are the degree of the Moon's apogee. Subtract minutes equal to (1+1/8) times the years elapsed. This will give the mean motion of the moon's apogee (Laghumanasa 1-5)

Multiply the number of years elapsed since the epoch by 10, then add to one eighth of itself and then add the sankranti tithi, put down the result in two places one below the other. In the lower place, deduct 1.60 of itself and divide what remains by 30. Deduct the remainder of this division from the result put down at the upper place. To that add the number of tithis elapsed (since the caitradi) from that subtract 3 times the number of years elapsed (since the epoch and also the number of seasons elapsed (since the caitradi of the current year) what is now obtained is Dvyuguna. This being divided by 7, remainder counted from the day for which epochal constant are computed gives current day (Laghumanasa) (*Dvyuguna thus calculated is used in other places for calculating various parameters of planets*)

Mathematical representation of the above explanation is as follows . Dyuguna = 10Y+10Y/8+St-R+Ct-3Y-Sl

R is again explained as (10Y+10Y/8=St)(1-1/60=30Q+R

Where Y = years elapsed since the epoch

St = Sankranti tithi; Ct = Caitradi tithi and S = seasons elapsed since the caitradi

A planet moves along its mean circular orbit at the rate of its mean motion, when it is above its mean orbit, it moves at slower rate and when below, at a higher rate (Sishyadhi Vruddhi Tantra 19-37)

From the sun at a distance of half a circle, moves thereon the shadow of the earth (Aryabhateeyam 4-2b)

A planet naturally moves to the east, but when retrograde, to the west It is drawn north or south by its declination Thus the motion of planets are of six kind (Sishyadhi Vruddhi Tantra 19-38)

If the planet rotates at a higher speed than its apogees, it is clockwise in its orbit and if lower speed, then it is anticlockwise motion (Aryabhateeyam 3-20)

The planet moving with equal linear velocity in their own orbits complete the circumference of the sphere of asterisms in a period of 60 solar years and - a distance equal to - the circumference of the sphere of the sky, in a Yuga (Aryabhateeyam 3-12)

All the mean planets move in their own orbits and the true planets in the eccentric circles *parimandalam* in shape. All the planets whether they move in their own orbits or eccentric circles move with their own mean anticlockwise from apogee and clockwise from their sighrocca apogee (Aryabhateeyam 3-17)

The rate of motion of the true planet in the eccentric, is the same as that of the mean planet in the eccentric, when the planet is at the points of intersection of the two circles, its mean motion is the same as its true motion (Sishyadhi Vruddhi Tantra 14-13)

The eccentric circles of each of these planets is equal to its own orbit, but the centre of the eccentric circle lies at a distance from the centre of solid earth. (Aryabhateeyam 3-18)

All the planets undoubtedly move with mean motion of the circumference of the epicycles. A planet faster than its uccha moves clockwise on the circumference of its epicycle and when slower than its ucca moves anticlockwise on its epicycles (Aryabhateeyam 3-19b, 29)

The mean planet lies at the centre of its epicycle, which is situated on the orbit (Aryabhateeyam 3-21b)

The retrograde motion of the planets beginning with Mars (Mars, Mercury, Jupiter, Venus and Saturn) commences when the sighroccakendras are respectively 163°, 145°, 125°, 165° and 113° When sighroccas are respectively 360° minus each of these values their retrograde motion ceases. (Sishyadhi Vruddhi Tantra 3-20)

It is said that the retrograde motion of the planets Mars, Mercury Jupiter, Venus and Saturn last for 66, 21, 112, 52, 134 days respectively (Sishyadhi Vruddhi Tantra 3-21)

The epicycles move anticlockwise from the apogees and clockwise from the sighrocca. The mean planet lies at the centre of its epicycle which is situated on the planets orbit (Aryabhateeyam 3-21)

Mars It is seen if its longitude is more than 15°, then within 188 days it moves through an angle of 60°, during next 108 days 60°, next 72 days 9° next 68 days 50°, next 240 days 70° it moves and then sets. (total days 768 and total degrees equal to 360°) (Pancha Siddhantika 17-69)

Mercury : When Mercury is behind the Sun by 12°, within 10 days it rises in the east. During 14 days it travels 10°, then for 18 days 9°, and it sets, for 30 days it travels for 13° and rises in the west, then for 18 days 9°, 16 days 8°, it travels and sets in the west. Then for 8 days it retrogrades by 9° and then reaches the starting point (Pancha siddhantika 17-71, 72)

To obtain the sighrocca of Mercury add at the rate of (4+6/65) degrees per day to it's druva (position at the end of the previous year) (Sishyadhi Vruddhi Tantra 1-36)

Jupiter . For 16 days it retrogrades and rises in the east, then for 54 days it travels for 44°, 70 days 64°, 109 days 120°, 88 days 76°, 40 days 32°, and then sets. The Jupiter travels for 16 more days for 12° prior to becoming niramsaka (0°) (Pancha Siddhantika 17-74, 75)

The mean longitude of the Jupiter is obtained by adding to its dhruva (at the end of the previous year) in minutes equivalent to 5 time the ahargana diminished by its 5/354 degree (Sishyadhi Vruddhi Tantra 1-36b)

Venus: For 5 days it is retrogrades and rises in the east, then for 15 days further retrogrades through an angle 21°, further for 208 days 15° retrograding, for 12 days 5° forward moves and sets. Then it travels for 10° during 48 days and becomes nirmasaka (Pancha siddhantika 17-77, 78)

EARTH

The spherical earth made of soil, water, fire and air, also is circular from all side (Aryabhateeyam 4-6)

Perfectly circular, perfectly spherical, made of wood, marked with degrees and minutes, incorporated with both longitude and latitude lines at ends. This is the golayantra. (globe Pancha siddhantika 14-23)

Made of wood, fully circular, uniform, equally dense throughout, and spherical shaped golayantra, which rotates at the rate of time by the help of mercury, oil and water, using our intelligence, is the Globe (Aryabhateeyam 4-22)

Naturally earth is spherical throughout. Due to the large area, peculiarities, etc, one does not feel that it is spherical. As we see only a small ups and downs on the surface of the earth. (Neelkanta Golapada 2-8)

During a day of Brahma the size of the earth increases externally by one yojana and during the night of Brahma, which is as long as

the day, this growth of the earth is destroyed (Aryabhateeyam 4-8)

Seven spheres are surrounding the earth. They are avaha, pravaha, udhwavaha, samvaha, suvaha, purvaha, paravaha. The avaha is the well known atmosphere of earth (Sishyadhi Vruddhi Tantra 17-1) Circumference of the avaha i.e atmosphere is 3375 Yojanas. The wise man to say that its diameter is 1074 yojanas (1 yojana = 12.11 kms.)

The spherical earth made of ether, fire, water, air and clay and thus have all the properties of five elements surrounded by the orbits and extending upto the sphere of stars remain in the space (Sishyadhi Vruddhi Tantra 17-1)

Earth surface is said to have an area of externally which is like a net enclosing a ball is equal to 2856338557 sq. yojanas (AD 749 Lallacharya's Sishyadhi Vruddhi Tantra 17-11)

The circumference of the earth is 3300 yojana. Its diameter is 1050 yojana. The mean circumference of the equinoctial shadow (palakarna) of a place give the corrected circumference at that place (Sishyadhi Vruddhi Tantra 1-44)

By using the above circumference of earth in yojana, it is found by means of simple proportion that the circumference of the circle of one revolution of a planet consists of 21600 minutes. Then can the earth be of infinite size? (Sishyadhi Vruddhi Tantra 20-32)

The earth rotates through an angle in the orbit, at the rate of 1 minute of an angle per 4 seconds (unit of time is prana = the time taken for one respiration = 4 Secs.) (Aryabhateeyam 1-6)

The rotation of earth cause of sidereal days (Aryabhateeyam 3-5).

The eastward revolution of the earth in one Yuga is 1582237500 (i.e on 4320000 years Aryabhateeyam 1-3)

Just as a man in a boat moving forward sees the stationary objects as moving backward, so are the stationary stars seen by the people at Equator (Lanka) as moving exactly towards the west. (Aryabhateeyam 4-9)

If earth rotates through one minute of arc in one respiration, from where does it start its motion and where does it go? And if it rotates

at the same place why do tall objects not fall down? (Brahmasphutasiddhanta 11-17)

If earth rotates, how could birds come back to their nest. Moreover arrows shot towards the sky would fall towards the west. (Sishyadhi Vruddhi Tantra 20-42)

If earth rotates to the east, the clouds would move to the west. If the earth moves slowly then how can it go (a full) round in one day (Sishyadhi Vruddhi Tantra 20-43)

Those who are at the distance half the earth's circumference from each other It is like the man-standing on the bank of river-and his reflection on the water, the sky is above all, the globe earth is beneath it. The inhabitants are on the earth.(Sishyadhi Vruddhi Tantra 17-5)

Just as house lizard resting on the ceiling of house, runs forward without hesitation so do people on the bottom side of the sphere earth. (Sishyadhi Vruddhi Tantra 17-7)

In our daily life-we see- the flame of fire goes towards the sky and heavy weight falls towards earth In the same manner everything that has a surface to reach makes for it. The earth however has no such surface, where can it fall? (Sishyadhi Vruddhi Tantra 17-8)

Whatever is staying in the space having mass, pulls other things towards itself with great force This gravitational pull keeps everything in the same position and also pulls others to fall in it. (Pancha Siddhantika)

The earth attracts all those solid materials in the sky towards it with it's own gravitational force. Those solid materials do fall on the earth. Since all the celestial bodies are attracted each other where can they fall? (Sidhanta Siromany, Bhuvanakosa, Goladhyaya 6)

The celestial sphere at the equator is constantly carried the west by the pravaha wind. Those who are in north pole it appears to move from left to right and those who are standing (by facing others in the south pole, it moves from right to left (Sishyadhi Vruddhi Tantra 18-3)

When it is sunrise in Lanka the same Sun sets in Siddhapura. It is noon in Yavakoti and midnight in Romaka desa (Aryabhateeyam 4-13)

When the Sun rises in Lanka, the abode of Rakshsasa it sets simultaneously in Siddhapura. When it is midday in Yavakoti, it is midnight in Romaka (Sishyadhi Vruddhi Tantra 17-12)

When it is sunrise in Lanka, it is said that Sun sets in Siddhapura (Brahmasphutasiddhanta 11-12)

Mathematicians say that one hundredth of the circumference of the earth appears to be plane. So, that portion of the earth appears to be plane to an observer (Sishyadhi Vruddhi Tantra 20-35)

To men the sky appears to meet the earth all round the horizon, So, the earth thus surrounded appears to be plane like a mirror (Sishyadhi Vruddhi Tantra 20-37)

North and South Pole of Earth

The gods living in the north at the Meru mountain (north pole) see one half of the Bhagola as revolving from left to right (clockwise) the demons living in the south at the Bhadvamugha-south pole-on the other hand see the other half as revolving from right to left in the anticlockwise. (Aryabhateeyam 4-16)

The gods from the north pole see the Sun after it has risen for half a solar year, so is done by demons in the south pole too. (Aryabhateeyam 4-17a)

In north pole there is no difference of direction (East, West, north or south). These directions are not decided by the Sun there. The Sun stands at a position in the pole The earth is rotating around it (Pancha siddhantika 15-11)

The arctic circle is known as Meru, it is only 1 yojana in area, covered by ice, it is in the midst of Nandanavana and full of gold (ratna) and it is perfectly circular (Aryabhateeyam 4-11)

Whom so ever proceeds from the north pole or south pole towards the equator he would observe the celestial sphere gradually rising and celestial pole more and more depressed from his zenith. The depression of the pole from the zenith being the same as the elevation of the celestial sphere. The number of degrees in the colatitude of a

place gives the corresponding length in yojana (Sishyadhi Vruddhi Tantra 18-6,7)

During the month of Mithuna Sun is seen at 24° to those who are at North pole To those who are at Avanti they can see the sun over their head. In north pole the day is maximum during mesha, vrushabha, and mithuna months, where as from the month of Karkitaka, the night increases. (Pancha Siddhantika 12-10)

Meridian, Latitude and Longitude

The vertical circle which passes through the east and west points is the prime vertical, and the vertical circle passing through the north and south points is the meridian. The circle which goes by the side of the above circles and on which the stars rise and set is the horizon (Aryabhateeyam 4-18)

The circle which passes through the east and west points and meets at distance is called the latitude (Aryabhateeyam 4-19 a)

The latitude lines are marked in degrees on the spherical surface of the (globe) earth (Brahma sphuta siddhanta 21-50)

The latitude which passes through Lanka is the one which passes through the middle of earth-equator- (Sishyadhi Vruddhi Tantra 16-12)

That line which exists and passes through Lanka, Vatsyapura, Avanti, Himalaya (place of Eeswara) and north pole (Suraalaya) is known is international longitude line (Laghubhaskareeyam 1-23)

From the centre of the land and water, at a distance on one quarter of the earth's circumference lies Lanka, and from Lanka at a distance of one fourth there of exactly northwards lies Ujjaini (Aryabhateeyam 4-14)

Avanti is to the north of Lanka at a distance of one fifteenth of the circumference of earth (Brahmasphuta siddhanta)

Fix a pole in Lanka, (equator) tie a thread in that, take it to the north pole tie it there also, then one can see the line of the thread passing through Lanka, Khara city, Kharapuri,.... and other cities. This is the international meridian line (Laghubhaskareeyam)

Thus (longitude) line has east and west sides on either side of the meridian line. Countries are marked based on this meridian, and the distance in yojana, that is the way for finding the longitude (Sankaranarayana, page 23)

Ascertain the shortest distance in yojanas between a place on the meridian line and the observer's station. Multiply the degrees in the difference of latitudes of two places by 3300 and divide by 360 Square the result, subtract it from the square of the yojana. Find the square root. This is the distance between the meridian lines passing through the two places. Multiply it by the mean daily motion of a planet and divided by the corrected circumference of the earth. The result in minutes should be added (to the longitude) for places to the west of the meridian line of Lanka and subtracted for places to the east. (Sishyadhi Vruddhi Tantra 1-44, 45).

By the distance in yojanas of the local place, east or west of the meridian of Avanti, multiply the 60th part of the degrees of the planet's daily motion, subtract the resulting minutes, from add them to the longitude of the planet (according as the place is east or west of the meridian of Avanti (Laghumanasa 4-3)

i.e. Longitude correction = + or - D m/60 min where D= distance in yojanas from Avanti, m = planets daily motion in degrees. The circumference of earth is taken here as 3600 yojanas.

When compared to the position of the meridian, people say that "I am standing on the east of it, west of it". Some people can also say that I am standing on the meridian line also (Sankaranarayana on Lakghubhaskareeyam 29).

The line drawn parallel (sama rekha) to the longitude/latitude in any country (my country-swadesha) and finding the difference from the meridian (angle) and doing the calculation, one can find out the distance on earth. And extrapolating this, one can get the celestial bodies position in sky also (Laghubhaskareeyam 1-25).

The time is calculated based on the meridian. Divide the time by 60 and calculate the longitude. Towards the east subtract and towards the west add the number (Laghubhaskareeyam 1-31)

The mean longitude are the places on the meridian passing through Lanka, ujjaini and Himalaya. When to these longitudes are applied correction for difference in terrestrial longitudes the result are for places east or west on the meridian (Sishyadhi Vruddhi Tantra 1-42)

As the Sun rises first at a place which is to the east of the prime meridian line and then at a place which is west. The correction for terrestrial longitude is applied positively or negatively as the case may be (Sishyadhi Vruddhi Tantra 16-6)

Multiplying the angle of the place from the meridian with the time unit and dividing by 60 one can get the distance of that place from the meridian (Sankaranarayana in Laghubhaskareeyam 32)

The latitude of the Mars, Mercury, Jupiter, Venus and Saturn when at their mean distance from the earth are respectively 9, 12, 6, 12 and .... .., each multiplied by 10, 4, 2,8, 6 and 10 each multiplied by 10 are respectively the degree in longitudes of the nodes of the above planets (Sishyadhi Vruddhi Tantra 10-5)

The Rsine of the latitude multiplies by the Rsine of the given declination and divided by the Rsine of colatitude gives the earthsine (Aryabhateeyam 4-26a)

The Rsine of true longitude of the Sun multiplied by the Rsine of 24° (according to modern science, it is 23½° and divided by the radius gives the Rsine of the declination. This multiplied by the equinoctial midday shadow and divided by 12 gives the earthshine or ku jya-ku=earth, jya=Rsin@ (Sishyadhi Vruddhi Tantra 2-17)

SUN AND MOON

The apogee of Sun is 78° (Sishyadhi Vruddhi Tantra 2-9)

The apogees of Sun is two sign and 18 degrees (Mallikarjuna sun 2-9)

Set down the Dyuguna in two places-one below the other - In the lower place add 10 times the years elapsed-since the epoch (the epochal position of the Sun, etc. are given elsewhere for saka era in this text) and divided by 70 Deduct the quotient from the Dyuguna put down at the other place; further subtract 8 time the years elapsed. Whatever is thus obtained is in degrees. To this add minutes equal

to 1/8 of the number of years elapsed. This will give the mean motion of the Sun since epoch. (Laghumanasa 1-3). This explanation is summarised as follows.

Sun's mean motion since epoch (epoch is any date universally fixed as the starting point) = D-(D+10Y)/70-8Y degrees + Y/°min. Where D is the Dyuguna, and Y years elapsed since the epoch.

Set down 13 times the Dyuguna in two places. In one place diminish it by 3 times the years elapsed and also by the Dyuguna. Divide that by 68 Add what is thus obtained as well as 24 times the years elapsed to the quantity put down at the other place- this will give the mean position of the moon since the epoch (Laghumanasa 1-4) It is summarised as follows.

Mean motion of moon = 13D+(13D-(D+3Y)/68+24Y degrees. D is Dyuguna and Y the years elapsed since the epoch.

When the mean distances are multiplied by their respective mandasphuta hypotenuse and divided by the radius, the results are their correct distance. Or the mean distance multiplied by the mean motion and divided by the true motion gives the correct distance of Sun and Moon from the earth (Sishyadhi Vruddhi Tantra 4-5)

The greatest altitude of the Sun is 24° (if viewed from the Sun, this is the declination for earth) Sishyadhi Vruddhi Tantra 18 - 1b)

The angular diameter of the Moon is obtained by multiplying the true motion with 11 dividing by 272. This value will be in minutes. The angular diameter of the sun is obtained by multiplying its true motion by 11 and dividing by 20. The difference between 8 times the true motion of the moon and 25 times that of sun divided by 60 gives the angular diameter of the shadow. (Sishyadhi Vruddhi Tantra 5-9)

7400 divided by the Sun's divisor is the diameter of the Sun's disc-in terms of minutes- and 3100 divided by the moon's manda divisor is the diameter of the moon's disc in terms of minutes. (Laghumanasa 6-3).

The Manda cheda-Manda divisors are constants which are 224, 97, 45, 100, 92, 320 and 63 for Sun, Moon, Mars, Mercury, Jupiter,

Venus, and Saturn respectively and corrected by the half the manda kotijya (Laghumanasa 3-3). The correction factor is 8°8′cos@/2.

8000 Nr make a yojana, the diameter of earth is 1050 yojana, of the Sun and Moon are 4410 and 315 yojana, of Meru is 1 yojana and the Venus, Jupiter, Mercury, Saturn and Mars at the moons mean diameter are 1/5, 1/10, 1/15/ 1/20, 1/25 of the moon's diameter (Aryabhateeyam 1-7)

When the sun is in the horizon, it is at a higher distance and its rays are dispersed by earth, it can be seen without discomfort and looks big and less hot (Sishyadhi Vruddhi Tantra 16- 46b, 47a)

Multiply the R sine of the sun's altitude for the given times by the Rsine of latitude and divide by the Rsine of colatitude. The result is the Sun's sankvarga (It is defined as the distance of the Sun's projection on the plane of the observer's horizon from the sun's rising and setting line) (Aryabhateeyam 4-29)

Divide the product of the Madhyaya (Rsine of the zenith distance of the meridian ecliptic point) and the udhayajya (Rsine of the amplitude of the rising point of the ecliptic) by the radius The square root of the difference between the squares of that result and the Madhyajya is the sun's or moon's parallax (drkshepa) (Aryabhateeya)

The number of revolutions of moon in a Yuga multiplied with 21600 gives the distance in yojanas. (I.e 12474720576900 yojana) i.e. circumference upto which the rays of sun reach in the sky (Mallikarjuna Suri 5-2)

Just as the sun rays reflected by mirror dispel the darkness in a room, the sun's rays reflected by the moon dispel the blind darkness of the night in the earth (Sishyadhi Vruddhi Tantra 16-39)

By what reason the vruddhikshaya or moon occurs? It is never the real change in the growth of destruction of moon! One who observes from the earth in the beginning of the lunar month. The area where the sunrays fall increase steadily day by day, and that area becomes easily visible by light (Sankaranarayana 4-4)

Multiply the ahargana with 38100 and subtract 1984 from the value, then divide by 1040953 to get the exact longitude of the sun (Pancha siddhantika 8-4)

Multiply the ahargana with 110 then add to this 609, divide the number with 3031 to get the centre of moon in Avanti, at the time of sunset there For completing 110 revolutions, the centre of moon (chandra kendra) takes 3031 days (Panchasidhantika 8-5)

At the end of lunar month-amavasya-lower half of the disc of moon visible to the people of this earth is, completely dark, but at the end of the first fortnight (purnima) it is completely white since the moon approaches the sun. In the dark half of the lunar month, it's dark portion gradually increases. However the moon recedes from the sun and so its illuminated portion gradually increases (Sishyadhi Vruddhi Tantra 6-38)

The difference in true longitude of the sun and moon in degree, multiplied by the radius of the moon and divided by 90 gives the illuminated portion of moon (in the light half) when the above difference is diminished by 180° and the remainder is multiplied by the radius of the moon and divided by 90°, the result is the dark portion in the dark half (Sishyadhi Vruddhi Tantra 9-13)

Ahargana multiplied by 78898 and divided by 2155625 gives mean longitude of the moon which is the same as the true longitude of the sun at sunset increased by 6 signs, the moon rises at the same as the sun sets, if greater, it rises after and if lesser it rises before sunset. (Sishyadhi Vruddhi Tantra 8-9)

ECLIPSE

What does it means that Asura is responsible for the eclipse? Others say that a snake Rahu swallows the sun and moon! Those are all puranic stories! Then what is called the Rahu (Sankaranarayana)

That is why it is said that the shadow of the earth is the cause for the lunar eclipse (Sankaranarayana 4 - 40 71)

The moon covers (shadows) the sun and the great shadow of earth covers the moon which causes the eclipse (Aryabhateeyam 4-37)

That portion in the space where the rays/ light is absent (i.e darkness) is called the shadow (Neelakanta golapada)

As the shape of the earth is spherical and it is placed in the centre of the space (bhapanchara madhya) the shadow of the earth having a circumference proportionate of the circumference of the earth moves in the opposite to the, direction of the sun's movement, with the same speed, similar to the shadow generated by a spherical vessel, and that is the shadow which can be seen from a distance (Sankaranarayana 71-72)

At the end of the lunar month the moon, which is nearer the sun as per degree measurement, enters (shadows) the suns disc similarly the end of the paksha the moon enters to the shadow of earth, which results in the eclipse (Aryabhateeyam 4-39)

As I know the elative position of the sun, moon and other asterisms, I say that the solar eclipse is taking place always at some place in the space. Due to the difference of place at some places they are seen (Pancha siddhantika 15-1)

If a line is drawn between the observer's eye and the sun, and if the moon happen to be in the line, whichever places the observers stand they can see the solar eclipse, this is happening any day and anywhere (Pancha siddhantika 15-3)

If a solar eclipse takes place during sunrise for us, the same eclipse may not be observed at the same time where the sunsets, so is the case where it is noon (Pancha siddhantika 15-8)

Those who are in the north pole and also near the place, they can never see the solar eclipse This is because the sun and moon never comes in the same line for them (Panchasiddhantika 15-5)

In the north pole the moon never comes in the line of the observers viewpoint. They always see the sun and moon at a difference only (hence the eclipse is not seen for them) (Pancha siddhantika 15-6)

In a solar and lunar eclipse, the parts of disc are different and duration are different, direction of contract and separation are different. The cause is the moon for solar eclipse and earths shadow in lunar eclipse. It is thus established that Rahu is not the cause for

eclipse (Sishyadhi Vruddhi Tantra 16-34)

The shadow is the moon's obscuring body and the moon is sun's obscuring body There are total and partial eclipses of the obscured body caused by the obscuring body The eclipse is named after the eclipsed portion of the - obscured body (Sishyadhi Vruddhi Tantra 5-10)

In solar eclipse since the moon comes from the west like a cloud and obscures the sun, contact takes place on the west and separation in the east (Sishyadhi Vruddhi Tantra 16-32)

In a solar eclipse the moon moving eastward enters the shadow which is moving westward, contact takes place on the east and separation from the west (Sishyadhi Vruddhi Tantra 16-30)

As the disc of earth's shadow-which is the obscuring body in a lunar eclipse- is big, the horns of moon, when half eclipse is blunt. The duration of eclipse is long and the obscured portion does not appear different from different places (Sishyadhi Vruddhi Tantra 16-33)

In a lunar eclipse, the moon enters the circle of the earth's shadow, in its orbit as shown by calculations. Since orbit is the same there is neither parallax in longitude nor in latitude (Sishyadhi Vruddhi Tantra 16-31)

The diameter of moon is 315 yojanas and that of the sun is 4410. The sun s distance multiplied by 5 and divided by 16 gives the hight of the cone of the earth's shadow (Sishyadhi Vruddhi Tantra 4-6)

The length of earth's shadow diminished by correct distance of the moon from the centre of the earth, multiplied by 1050 and divided by itself gives the result, the diameter of the earth's shadow in yojanas (Sishyadi vruddhi Tantra 5-7)

The longitude of the sun plus 6 signs (180°) is the longitude of the shadow of planet. It's diameter in terms of minutes, in the moon's orbit, is equal to 8300 divided by the moon's divisor (Laghumanasa 6-4) i.e Diameter of shadow 8300/(97°+8°8′cos@/2) @ is moons bhuja.

Subtract the longitude of the moon's node from that of sun Find the difference between this remainder and for 12 signs-180° or 360° divide the result expressed in minutes by the sum of motions of sun and the node The quotient gives the number of days elapsed (since

the possibility of the occurrence of an eclipse and if the above days elapsed (since the possibility of the occurrence of an eclipse and if the above days are greater than for 12 signs, if less) the quotient gives the number of days to elapse (before eclipse) if these days happen to be near amavasy then there is a possibility of solar eclipse during day time. If they happen to be near purnima, then is possibility of lunar eclipse during night. Subtract the longitude of moon's node from that of the moon. If the difference between the remainder and 12 sign lies within 12° There is possibility of lunar eclipse. (Sishyadhi Vruddhi Tantra 7-1,2)

Multiply the distance of the sun from earth by the diameter of earth and divide the product by the difference between the diameters of sun and earth; the result is length of the shadow of earth from the diameter of earth (Aryabhateeyam 4-39)

Multiply suns true motion by 5 and divide by 28. Multiply the moons true motion by 2 and divide by 35. The difference of two quotients gives the diameter of earth's shadow in angulas (Sishyadhi Vruddhi Tantra 7-4)

Multiply the difference between the length of earth's shadow and the distance of the moon by the earth's diameter and divide the product by the length of the earth's shadow; the result is the diameter of the Tamas shadow in moon's distance (Aryabhateeyam 4-40)

In a lunar eclipse the moon is said to be in the eastern hemisphere from midday till midnight. From midnight til midday it is said to be in the westen hemisphere. In a solar eclipse the contrary is the case. (Sishyadhi Vruddhi Tantra 5-24)

At the beginning and end of the eclipse the moon is of dense smoky colour, in a partial eclipse. It is always dark as a mass of collyrium, when the obscured portion is greater than half it is dark red. When it is completely obscured it is tawny (Sishyadhi Vruddhi Tantra 5-36)

From the square of half the sum of the diameters of that shadow and the moon, subtract the square of moon's latitude and take the square root of the difference, result is known as the duration of eclipse in terms of minutes of arc. The corresponding time is obtained with

the help of daily motion of sun and moon. (Aryabhateeyam 4-41)

When 1 12 of the sun is obscured, the eclipse cannot be seen because of the brightness of sun Owing to the moon's clearness even 1 16 part of it when eclipsed can be seen (Sishyadhi Vruddhi Tantra 6-17)

Some say that if the eclipsed portion of sun is less that 1/8, then eclipse cannot be seen (Sankaranarayana)

If the overlapping sun disc (by moon) extends only less than 1/8, then eclipse need not be predicted This is because of the brightness of sun which makes the eclipse non visible (from the earth) (Aryabhateeyam 4-47)

Mark the three extremities of the latitudes (at the beginning, middle and end of eclipse) draw two fish figures- one passing through the first two points and the other through the last two points Draw two lines passing through the mouth and tail of each fish figure (with the point of intersection of these two lines) draw a circle passing through the three extremities of the latitudes from the centre (of the concentric circular) just touching the path (Sishyadhi Vruddhi Tantra)

The observer on the surface of earth sees the disc of the sun obscured by moon even before-the calculation time for conjuction- as he is elevated above the centre (this is so if the sun is) in the Eastern hemisphere

But the sun, in the western hemisphere, he sees it after the calculated time, when the sun has set, that is it has disappeared below the horizon So the parallax in longitude due to the radius of the earth is subtracted from the calculated time of conjuction-if the eclipse takes place in the eastern hemisphere and added if it takes place in western hemisphere. (Sishyadhi Vruddhi Tantra 16-24,25)

The line joining observer at the centre of the earth and the zenith coincides with the line joining observer on the surface of earth to the zenith. Thus there is no parallax at the mid day (Sishyadhi Vruddhi Tantra 16-26)

Whatever may be the reasoning given by the astronomers for the parallax in longitude in minutes due to eastern and western horizons,

similar reasoning is to be understood for the parallax in latitude in minutes due to the northern and southern horizons. Or the parallax in latitudes is calculated by means of the degrees in the zenith distance of the meridian ecliptic point. It is the north south distance of the orbit of sun and the moon at the time of an eclipse. (Sishyadhi Vruddhi Tantra 16-27, 28)

Subtract 26′ from the longitude of the Rahu (shadow). Take the angle difference between the Rahu and the moon. If this comes within 13° there is a possibility of eclipse. If it is within 15°, it becomes only possibility (only grahana chaya occurs) (Pancha Siddhantika 6-2)

If one wants to know there will be an eclipse after 6 months find the mean longitude of the sun, moon, their apogees and nodes on that day. Then one must add to the first three longitudes 5s 24° 27′6″, 5s22°12′53″, 0s19°42′53″ and subtract 0s9° 22′ 41″ from the longitude of the node (Sishydhi Vruddhi Tantra 7-9,10)

If one wants to know whether there was an eclipse before half a lunar month or there will be one after half a lunar month, one must first find the mean longitude of the sun, its apogee and node on that day Then one must add or subtract 0s 14°47′2″,. 6s17°38′42″ and 100′13″ respectively to or from the first three mean longitudes according as the eclipse is after or before half a lunar month (the process should reverse in the case of the moon node) 47′45″ This is the rule for determination of an eclipse (Sishyadhi Vruddhi Tantra 7-11)

[illegible]

The position of the stars are said to be like this by the ancient r sages This is in the order of aswini star onwards· 48′, 40′, 56′, 54′, 52′, 20′, 72′, 70′, 44′, 48′, 36′, 44′, 78′, 66′, 62′, 72′, 52′, 8′, 6′, 4′, 4′19′, 18′, 40′, 42′, 12′, and 74′, each multiplied by 10 They also say that the polar longitudes of each nakshtra is equal in minutes to the sum of its position and 800 times the number of the star proceeding it - total 27 star constellation (i.e its own number in the above order) (Sishyadhi Vruddhi Tantra 11-1-3)

The polar latitudes of the stars from aswini, measured from the ends of their respective declination on the ecliptic corresponding to

their polar longitudes are: 10°, 12°, 5°,5°, 10°, 11°, 6°, 0°, 7°, 12°, 13°, 8°, 2°, 37°, 1°30′, 3°, 4°, 8°30′, 5°20′, 5°, 30°, 36°, 20′, 24°, 26°, 0°, (1°=24 angulas) the polar latitudes of Agastya is 2s 27° i.e 87° x There is conjuction of the planets with Agastya. (Sishyadhi Vruddhi Tantra 11-4-7)

The latitudes of ardra, jyesta, utrashada, poorvashada, rohini, mula, hasta, anuradha, citra, satabhasha, mrgasira, visakha are to the south. The latitudes of pushya, revati and magha are 0° i.e they are seen at the end of their respective declinations. The remaining stars have northern latitude. (Sishyadhi Vruddhi Tantra 11-9,10)

If the true longitude of the planet is greater than the polar longitude of the star their conjuction has taken place, if less, it will take place. When retrograde the contrary is true (Sishyadhi Vruddhi Tantra 11-4c, d)

COMETS

Some people say that the number of comets is 100, others say it is 1000, but sage Narada says that the same comet is appearing in different ways, at different places and times (Bruhatsamhita 11-5)

Maximum time period to observe a comet is 36 yrs, and average time 24 years and the minimum time 13 years for appearing (Bruhatsamhita 21-42)

There are hundreds of comets which appear again and again (Bhadrabhahu samhita 21-4)

x- When the true longitude of a planet in the same as this.

MATHEMATICS

A few quotations, in the form of exact English translation is given below The aim of selecting the sample quotations are to show the depth to which the ancient Indian mathematicians could go, in handling the modern mathematics.

Some quotations from the Sulba sutras (B.C 1300-B.C 700)

Diagonal of a square, produces double the area of the square itself (Boudhayana Sulba sutras 1.9)

The areas produced separately by the length and breadth of a rectangle together equal to the area of the (square) produced by the diagonal (Boudhayana Sulba sutras1 12)

(Example of the above rule) This is observed in rectangles having sides 3 and 4, 12 and 5, 7 and 24, 12 and 35, 15 and 36 (Boudhayana Sulbam 1.13)- hypotenese obtained as numbers without fractions- With half unit of the side of a square 1/4 unit area is produced (Apastamba Sulbam 3.10)

The bricks to be made by four sides having the measurements 1/8 purusha, 3/8 purusha 1/4 purusha and square root of 1/4 prursha are used (Apastamba Sulbasutram 19.2)

The sacrificial hut pragvamsa occupies on the ground a square area of side 10 aratnis, the hut for the (wife) patnusala, a square with area 4 aratnis, end of the vedi, is at a distance of 3 prakrama from the pragvamsa and the hall called sadas is one prakrama away from the western end of the mahavedi. The sadas is 9 prakarama in area. The havirdhana of 12 prakramas is 4 prakrama from the sadas and 10½ prakrama from the yupavata. One pada is allowed for the yupavata and the remaining belongs to the Uttaravedi. The agnidra hut is a square of 6 aratnis. The chord measures 36 prakramas (Manava sulbasutra 3.1.3.3)

Fifth part of the diameter added to three times the diameter gives the circumference of the circle (Manavasulbam 11.13)

Divide the diameter of a circle into 10 parts and leave out three parts. The square drawn with this length as side and placed within the circle just project inside (Manavasulbam 11 14)

Four kinds of bricks are prepared with one third and one fourth (of a purusha) in area. They are one ninth of the original of (40, 40). Triangular (30, 30, 30, $\sqrt{2}$), haf triangular (15 $\sqrt{2}$, 30, 15 $\sqrt{2}$) and five cornered bricks (15 $\sqrt{2}$, 15 $\sqrt{2}$, 15, 30, 15) Having the measurements (given in parenthesis) for sides of bricks (Manavasulbam 14.21)

Some quotations of mathematical problems given by Bhaskaracharya I (AD 629) in his commentary for Aryabhatteeya (INSA publication) page and problem number in the parenthesis.

Tell me separately the cubes of integral numbers beginning with 1 and ending in 9 and also the cubes of $(8x8)^2$ and $(25^2)^{12}$ (51.4)

If you have a clear understanding of cubing a number say correctly the cubes of 6,5,10 and 8 as respectively diminished 1/6, 1/5, 1/10 and 1/8 (51.4)

Calculate in accordance with ganita of Aryabhatta the square root of 6+1/4 and of 13+4/9 (52.2)

Correctly state in accordance with the rule prescribed by Aryabhatta the cube root of 8291469824 (54.2)

Tell me the areas of three equilateral triangle whose sides are 7, 8 and 9 units respectively and also areas of the isosless triangle whose base is 6 unit and lateral sides each 5 units (55 1)

What is the area of the scalene triangle in which one lateral side is 13 units, the other 15 units and the base 14 units (56.3)

The diameter of 3 circles are correctly seen by me to be 8, 12 and 13 units respectively. Tell me separately circumference and area of the circle (60.1).

The base of the trapezium is 14 units, the face 4 units and the lateral sides each 13 units give out the junction line and area of the figure (63.1)

Calculate the diameter of the circle whose peripheries are 3299 minus 8/25 and 21600 respectively (76.2)

Shadow of the gnomon situated at a distance of 50 angulas from the foot of a lamp post is 100 angulas. Say what is the height of the lamp (92.3)

A full lotus blown of 8 angulas is seen just above the water, being carried away by the wind. At what distance it will just cover in the water level (101.6).

In an arithmetical progression series the first term is 2, the common difference is 3 and number of terms is 5. Tell me the middle term and the sum of the series (105.1)

In the month of Kartika, a king daily gives away some money starting with 2 on the first day and increasing that by 3 per day. 15 days having passed, there arrived a Brahmana well versed in Vedas. The amount for the next 10 days was given to him and that for the next 5 days of the month to someone else. Say what do the last two persons get (106.4)

There are (3 pyramidal) piles having 5, 4 and 9 cuboidal layers. They are cuboidal bricks with one brick in the topmost layer. Find the number of bricks used in them (111.2)

(Sum of $1^2+2^2+3^2+4^2+$ ......)

I do not know the interest on 100, but I do know that the monthly interest on 100 + interest accruing in 4 months is 6. Give me the monthly interest rate on 100 (114.1)

The monthly interest on 25 is not known. But the monthly interest on 25 rupas lent out elsewhere at the same rate of interest is seen to amount to 3-1/5 rupas in 5 months. I want to know the monthly interest on 25 rupas as also the interest for 5 months on the interest of 25 rupas (115.2)

A serpent of 20 cubit long enter into the hole moving forward at the rate of ½ of an angula per muhurta (unit of time) and backward at the rate of 1/5 of an angula per muhurta. In how many days does it get into the hole completely (118 4)

Five merchants collaborate (in a business). The capitals invested by them are 1 and the same number increasing successively by one (i.e 1, 2, 3, 4 and 5) The profit accrued amounts to 1000, say what share of profit should be given to whom (119.6)

The combined profit of three merchants investmented are in the ratio of ½ 1/3 and 1/8 respectively, amounts to 70 minus 1 What is the share of profit (119.7)

A number is multiplied by 2, then increased by 1, then divided by 5, then multiplied by 3 then diminished by 2, and then divided by 7, the results is 1 say what is the number (124.1)

One man goes from Valabhi at the speed of 1½ yojana a day. Another man comes along the same route from Harukaccha at the speed of 1 1 4 yojana a day The distance between the two places is known to be 18 yojana. Say after how much time will they meet each other (131.1)

A number leaves 1 as remaineder when divided by 5 and 2 when divided by 7 Calculate the number (133 1)

Other Quotations

The product of perpendicular and half the base gives the measure of the area of a triangle (Arybhateeya 2 6)

Half the circumference multiplied by the semi diameter certainly gives the area of circle (Arybahateeya 2.7)

The results obtained by multiplying half the sum of the base and the face by the height is to be known as the area of a trapezium (Arybhateeya 2.8)

The chord of the 1/6 the of the circumference of a circle is equal to the radius of the circle (Arybhateeya 2 9)

Hundred plus four multiplied by eight and added to 62000, this gives the near approximate measure of the circumference of a circle whose diameter is 20,000 (Arybahateeya 2.10)

In a circle (when the chord divided it into two arcs) the product of the arrows of the two arcs is certainly equal to the square of half the chord (Arybhateeya 2.18b)

(When one circle intersects other circle) multiply the diameters of two circles each diminished by the erosion, and divide the sum of the diameter of the two circles after each has been diminished by the erosion, then are obtained the arrows of the arcs (of the two circles) intercepted in each other (Arybhateeya 2.19)

Mathematics find it's use in all the social, ritualistic, spiritual and commercial purposes. It is used in the science of sex, economics, music, dramas, culinaries, medical educations, architecture, and so on. He continues to say that without mathematics nothing can exist (Ganita sara sangraha 1-16)

Why to tell much? Whatever exists in all the three worlds, all those things cannot exist without mathematics. (Ganita Sara Sangraha 1-17)

Just like a woman, even though she is one (and the same) she may be known as mother, daughter, and daughter in law. In the same way the number at the unit place carries unit value, at the tenth place ten times the value and hundredth place hundred times the value. I.e. same number carries different values at unit, 10th..... place in number (AD 600 Vyasa bhashya of the Yoga Sutra (III-13)

History of discovery and use of zero '0' in mathematics. It is now generally accepted the circular symbol for a zero as a part of the numerical system is an Indian contributions. Pingalacharya's Chandasastra (BC 200) appears to be the first book where the concept of *sunya* is appearing. Paulisa siddhanta and Suryasiddhanta also use the *sunyas* as a numeral like in the bhootha sankya *kha* (space-sunyaakasa) is also sunya which is equal to zero. Atharvaveda recension which is commonly followed by Kashmiri pandits, used circular dot for sunya to give the folio and page numbers from about 100 BC. In the Bakhshali manuscripts, which dated back 100 AD, the use of 0 was very common. Increase in the value of the numbers by the addition of zeros are defined in the book Manasollasa (2-97) written one millennia and a half ago.

From 1 to 9 numerals are of one digit. From ten onwards, steadily value increases by the addition of dots (zeros). (Siddhanta sekhara 14-6)

Whether it is a positive number or a negative number, value will not change when 0 is added or subtracted from it. But when the number (positive or negative) is subtracted from the 0, the positive number becomes negative and negative becomes positive. When multiplied with 0 the value of all the number becomes 0 and when divided, the value become infinity (Bhaskaracharya II)

In the infinity, whenever numbers are entered (added) or they are removed (subtracted) at any number of times, nothing will be happening (the value will remain constant). It is just like, during pralaya time, everything gets dissolved in Achuta (Mahavishnu) and after pralaya all those things come out. Even then nothing happens to Mahavishnu i e, when added to or subtracted from the value (shape and size) remain constant in INFINITY (Beejaganita)

Normal interest can be charged as 1⅓% per mensem, where as for the commercial loans 5% per mensem can also charges. (Vishnu smruthi 6.2)

2, 3, 4 or 5% can be charged as interest per mensem depending upon the caste of the borrower Sage Vyasa says that from common man more than 2% interest should not be charged (Vishnu Smruthi)

Pasam, ankusam, serpent, damaru, kapalam, soolam, khatvangam, sakti, saram, Chapam, are the 10 Bhooshanas of Mahadeva holding in 10 hands. If these items are hold in ten hands of lord Siva, tell how many permutations and combinations (figures) are possible for Mahadeva. Similarly tell the number of possible figure of Lord Vishnu if his four hands have Shanku, Chakra, Gadha and padma, and various combinations are drawn, with these items in different hands (Leelavati)

(Answer is that Lord Siva can have 10! (Factorial 10 i.e 1x2x3x4x5x6x7x8x9x10) number of figures if each items are mutually replaced by the other to find out different permutations and combinations Similarly Vishnu can have 4! (Factorial 4 = 1x2x3x4) number of combinations.

Having 5 digits, whose total is 13, how many numbers can be written by combining the digit, without using 0. (Leelavati)

From the total number of bees present in a bee hive, if the square root of half has gone to the garden, 8/9 of the total followed the first group. One bee got trapped in a flower, in the night and yet another bee happened to be nearer the one got trapped in the flower. My dear daughter tell me how many bees were there in the beehive? (Leelavati)

Multiply half of circumference of the circle with diameter to get area of the circle. (Aryabhateeyam)

The diameter multiplied with 4 and divided separately by the odd numbers 3,5,7 . . . And the results, alternatively subtracted and added to the extent required and the result is subtracted from 4 times the diameter. This gives accurate value of the circumference of the circle having the above diameter i.e

Circumference of a circle = 4D-4D(1/3-1/5+1/7-1/9+.....)

(Where D is the diameter of the circle) (Mahaveeracharya in Kriyakramakari in the section of Circles II-40)

Following are different theorems from the KARANAPADDHATI book of Puthumana Somayaji (AD 1438)

Four times of diameter is divided separately by each of the odd integers 3, 5,7,.. . Every quotient whose order is even, is taken away from the one proceeding it. Combined result of all such small operations when subtracted from four times the diameter, gives the value of the circumference with progressively greater accuracy

Circumference of a circle = 4D (1/3-1/5)-4D (1/7-1/9). ... Or II/4=(1-1/3+1/5-1/7+1/9.. ..) (Karanapadhathi 6-5)

Six times the diameter is divided separately by the square of twice the square of even integers 2, 4, 6, minus one, diminished by the squares of the even integers themselves. The sum of the resulting quotient increased by thrice the diameter is the circumference.

Circumference = 3D- $(6D/(2x2^2-1)^2-2^2+6D/(2x4^2-1)^2-4^2+6D/$ $(2)x6^2-1)^2-6^2+....$ (Karanapaddhati 6-4)

Multiply the diameter of a circle with 4 and keep it at different places and divide each with the cube of odd numbers beginning from 3,5,7, subtracted by the same value. Repeat this and add and subtract the results alternatively, to three times the diameter of circle to get the circumference to the highest degree of accuracy.

$3D+4D/(3^3-3)-4D/(5-5)+4D/^3(7-7)...$ = circumference. (Karnapaddhati 6-6)

The chord of an arc of a circle is obtained when the cube of arc length divided by 6 times the cube of the radius and the result is subtracted from the arc. (When @ is the angle)

Chord (R sin@ = $s - s^3/6r^3$. Where 's' is the length of the arc and 'r' is the radius of the arc. (Karnapaddhati 6-19)

The arc is multiplied by itself (for a definite number of times) and the product is multiplied once again by the same arc. The result is divided by the product obtained by multiplying 2,3,...etc (upto the same definite number) with the radius repeatedly multiplied by itself (the same number of times) The quotients thus obtained for the even values of (definite) numbers (upto which the aforesaid multiplication is repeatedly done) are set one below the other (in one column) and likewise those corresponding to the odd values of (definite) numbers (in another column). From the first terms is subtracted the term immediately below and so on for every column. All the values thus obtained in the case of the even column are subtracted together from the arc and the corresponding values for the add column together from he radius The results are the bujajya (Rsin@) and kotiya (R cos@), respectively In a nutshell this means the modern theorem resulting in the two following values (Karanapaddhati 6-12, 13a)

$\sin @ = @ - @^3/3! + @^5/5! - @^7/7! + \ldots$..And

$\cos @ = 1 - @^2/2! + @^4/4! - \ldots$ .. (where 3! etc. is factorial 3)

Chord (Rsin@) of the arc is to be multiplied by the semi diameter and divided by the koti (Rcos@). This is the first term (of the series). The value of the first term when multiplied by the square of the Jya and divided by the square of the koti gives the second term, This process is repeated. The successive terms are divided by the odd integer 1,3,5,.... Now, when the consecutive terms in the series starting from the first term are alternately subtracted and added gives the circumferences. This explanation can be summarised as follows.

$@ = \tan@ \; 1/3 \tan @^3 + 1/5 \tan@ \ldots$ Where the arc = r @.... (Karanapaddhati 6-18)

R Cos @ when multiplied with radius and diminished the result from the square of the radius and taken the square root of half of this gives the R sin@/2. (instead of diminishing) when added the results to the square of the radius and the processed as above the R cos @/2 will be obtained (Karanapaddhati 6-11)i.e

Square root of $(R^2 - R^2 \cos@)/2$ = Rsin @/2

Square root of $(R^2+R^2 \cos@)/2$ = R Cos @/2

The square root of the difference of the R sine 90° and R sine of an arc is R cosine of the arc or R sine of the difference of 90° and an arc is the R cosine of the arc (Sishyadi vruddhi Tantra 3-3)

Reduce the arc to minutes and then divide by 225 the quotient denote the number of Rsine differences to be taken completely, then multiply the remainder by the next Rsine difference and divide by 225. Add the quotient to the sum of the R sine difference obtained before The sum thus obtained is the R sine of the arc (this rule for finding out Rsine of any angle from the Rsine table given, for an interval of 225′ Upto the angles 90° of angles as early as AD 499 by Aryabhatta) (Mahabhaskareeya 4-3,4)

Subtract the degrees of the bhuja or koti (in angles @ from the degrees of half a circle (i e180°). Then multiply the remainder by the degrees of the buja (Rsin @ or koti (Rcos@) and put down the result at two places. At one place subtract the result from 40500. By one fourth of the remainder divide the result at the other place as multiplied by the antyaphala. Thus obtained the entire bhahuphala or kotiphala. This explanation is mathematically represented as

Sin@ = 4 (180-@) @/40500-(180-@) @ where @ is the angle (Mahabhaskareeya 7-17to 19)

For finding the arc corresponding to a given sine (value of an angle), find the residue left after subtracting as many as possible of the tabular differences of sines, multiply it by 900 and divide by the tabular difference to be passed over, by means of the minutes of arc obtained, find the true tabular difference by repeating the process and thus find the minutes of arc corresponding to the required residue of the sine. (Khandakhadyaka 9-14)

When the table for R sine@ is given as in Aryabhateeya book) If the given sine lies between Sin X and Sin X+C then the arc will be X+@ where @ is residual arc.)

Multiply the diameter with 4 divided by 1 keep the result apart. Again multiply the diameter with 4 and divide separately by the odd numbers 3,5,7,.. then subtract and add alternatively tot he first value (kept separately) to get the circumference of the circle and follow as given below (Tantrasamgraha 11)

Circumference = 4D/ 1-4D/ 3+4D/ 5-4D+... Or

Circumference /diameter = II = 4 (1-1/3+1/5-1/7+)

The diameter multiplied by 16 increased by 16 and divided by 113, the result is combined with trice the diameter to get the circumference of the circle. This result is more accurate than the accurate result obtained by other methods. (AD 816 Virasena in Dhavalateeka)

When half of the trijya (radius) is taken it is the chord of the arc 30° (Rsin 30). The square root of half of the square of radius is taken it is the chord of 45° (Rsin 45) From these two chord (Rsin@ of other angles can be calculated (Karanapaddhati 6-8)

When R = 3438′ (angular radius); Rsin 30 = 3438/2=1719′

R sin 45 = $\sqrt{3438^2/2}$ = 3438/$\sqrt{2}$ = 2431′

Half of the square root of the sum of the squares of the chord of an angle and its arrow gives the Rsine of half of the angle of the chord (Karanapaddhati 6-10a)

i.e R sin 15= ½ $\sqrt{(Rsin30)^2 + (R\ verse\ 30)^2}$= 890 (Rsin 30=1719′)

The square root of the difference of the square of the chord and that of the square of trijya (radius) gives the kotjya (Rcos@) (Karnapaddhati 6-10b) i.e $\sqrt{R2-(Rsin@)^2}$ = R cos@

i.e Circumference = 3D + (16D+16) / 113

1 Rsin30 = R/2   2. R sin60 = $\sqrt{3}$/2R

3. R sin 90=R   4. $(Rsin@)^2$ = ½R(R-Rcos2@)

5. $R^2 \sin^2 @^2 + R^2 Cos^2 @^2 = R^2$ (Panchasiddhantika of 4, 5, 19)

The square root of 1/8 th part of the square side of a quadrilateral is *hara*. *Hara* less 1/4th of the side is multiplied by the side and (the product is) divided by hara. Taking segments equal the result thus obtained along a side from its corners (i.e taking segements equal in length to this result on every side from the corners on it) the octagon is formed.

Adding the square of half of a side of the octagon to the square of radius, the square root of the sum obtained is the diagonal. By that diagonal is divided the square of the radius less half the square of a side of the octagon. The result is subtracted from the diagonal and difference is halved to get what is call the hara. The diagonal less held the diameter is multiplied by half of a side of the octagon, the product divided by hara gives along sides from corners and cut of to get a regular polygon of 16 sides. In this way of cutting portion of each side ultimately a circle can be obtained. (This is now known as the polygonal approximation to circles involving the construction of regular polygon of sides n) (Madhavacharya's Kriyakramakari)

Few examples of highly complex mathematical formula and equations taken from A History of the Kerala School of Hindu Astronomy By Dr. K. V. Sarma, are given

1. Tycho Brahe's Reduction to the ecliptic

pātonasya vidhos tu koṭibhujayor jīve mithas tāḍayet
antyakṣepaśarāhatam vadham amum vikṣepakoṭyā haret I
labdham vyāsadaloddhṛtam himakare svargam, vipāte vidhau
yugmāyugmapadopage, vidhur ayam spaṣṭo bhagole bhavet II

'Multiply the tabular cosine (*koṭijya*) and sine (*bhujajya*) of the moon-minus-node and the product by the tabular versine (*sara*) of the maximum latitude (*antya-kṣepa*) of the moon. Divide this by the tabular cosine of the latitude at the particular moment and the quotient is to be divided again by the tabular radius (*vyasadala*). The result (will give the correction for longitude which) is to be added to or substrated from the moon's longitude, as the moon minus-node is in

an even or on odd quadrant, respectively. The true Moon measures on the ecliptic is thus got'.

Acyuta's this formula may be expressed in terms of modern mathematics thus: If F is the longitudinal difference between the node and the planet, *w* the maximum latitude and *y* the actual latitude, them, the correction k = sin F. cos F (1-cos *w*)/cos *y*

2. Newton-Gauss Interpolation formula

gacchad-yāta-guṇāntaravapur yātaiṣya-diṣvāsanac-
chedābhyāsa-samūha-kārmukakṛti-prāptāt tribhis tāḍitāt I
vedaiḥ ṣaḍbhir avāptam antyaguṇaje rāśyoh kramād antyabhe
gantavyahāta-vartamana-guṇajāc cāpāptam ekādibhiḥ II
antyād utkramataḥ krameṇa viṣamaiḥ saṅkhyāviśeṣaiḥ kṣiped
bhaṅktvāptam, yadi maurvikavidhir ayam makhyāḥ kramād vartate
śodhyam vyutkramatas tathākṛtaphalam .....

(Bhasya on Mahabhaskariya, 4.22)

'Multiply the difference of the last and the current sine differences by the two parts of the elemental arc (made by any intermediary point on it) and divide by the square of the elemental arc and further multiply by three Now divide the result so obtained by four in the first *rasi*, or by six in the second *rasi*. The final result thus obtained should be added to the portion of the current sine difference (got by linear proportion)

In the last (third) *rasi*, multiply the linearly proportional part of the current sine difference by the remaining part of the elemental arc and divide by the elemental arc. Now, divide the result (so obtained) by the odd numbers (1, 3, 5 etc) according as the current sine difference (is first, second, third etc.) when counted from the end in the reverse order. Add the final result thus obtained to the portion of current sine difference (got by ordinary proportion). These are the rules for computing true sine differences for (direct) sines. In the case of versed sines, apply the rules in the reverse order and the above corrections are to be substracted from the respective differences (got by linear interpolation).

Using the general functional notation and finite difference operator, the rule for the second *rasi* may be put as.

$$f(x+nh) = f(x)+n\,\Delta f(x) + n(n-1)/2\ \{\Delta f(x) - \Delta f(x-h)\}$$

Which is a particular case (up to the second order) of the general Newton-Gauss interpolation formula.

3. Taylor series for Sine and Cosine functions

The approximations for sine and cosine functions up to the second power of small quantities, following from a well-known series expansion due to the British mathematician Brook Taylor (1685-1731) may be expressed as:

$$f(x+\theta) = f(x) + \theta f'(x) + \theta^2/2! \text{ unto} = x\ f''(x) \ldots$$

(approximately, when θ is small)

This approximation has been anticipated, in its particular cases, $f(x) = \sin x$ and $f(x) = \cos x$, by more than three hundred years, by Madhava of Sangamagrama (c. 1340-1425) in the following verses:

iṣṭa-doḥkoṭidhanuṣoḥ svasamīpasamīrite I
jye dve sāvayave nyasya kur yād unādhikam dhanuḥ II
dvighna-tallıptıkāpaikaśaraśailaśıkhīndavaḥ I
nyasyācchedāya ca mithas tatsamskāravidhitsayā II
chitvaikām prakṣipej jahyāt taddhanuṣyadhıkonake I
anyasyām atha tām dvighnām tathā syam iti samskṛtıḥ I
iti te kṛtasamskāre svaguṇau dhanuṣas tayoḥ II

'Placing the (sine and cosine) chords nearest to the arc whose sine and cosine chords are required, get the arc difference to be subtracted or added. For making the correction, 13,751 should be divided by twice the arc difference in minutes and the quotient is to be placed as the divisor. Divide the one, (say sine), by this (divisor) and add to or subtract from the other (i.e., cosine), according as the arc difference is to be added or subtracted. Double this (result) and do as before (i.e divide by the divisor). Add or subtract the result (so obtained) to or from the first sine or cosine to get the desired sine or cosine chords'.

4. Newton's power series for the Sine and Cosine

In Western mathematics, Newton (1642-1727) is credited with the enunciation about 1670, of the sine and cosine power series which might be stated as:

$$\sin x = x - x^3/3! + x^5/5! - \ldots.$$

$$\cos x = 1 - x^2/2! + x^4/4! - \ldots.$$

These formula are implied in the following verses of Madhava, depicting the derivation of the series of sine and tabular versine (*sara*) values of the arc correct to 1/3600 of a degree:

nihatya cāpavargeṇa cāpam tattatphalāni ca I
haret samulayugvargais trijyāvargahataiḥ kramāt II
cāpam phalāni cādhodho nyasyoparyupari tyajet I
jīvāptyai, sangraho syaiva vidvān-ityādinā kṛtaḥ II
nihatya cāpavargena rupam tattatphalāni ca I
hared vimūlayugvargais trijyāvargahataiḥ kramāt II
kintu vyāsadalenaiva dvighnenādyam vibhājyatam I
phalāny adhodhaḥ kramaśo nyasyoparyupari tyajet II
śarātyai, saṅgraho' syaiva stenastri-tyādinā kṛtaḥ I

'Multiply repeatedly the arc by its square and divide by the square of the even numbers (2, 4 etc.) increased by that number and then multiplied by the square of the radius. Place the arc and the results (of the above operation) one below the other and subtract each from what is above it. To derive the arcs, which are collected (and stated, in order, in the mnemonic verse) beginning with the expression *vidvan* (i.e. 0° 0′ 0″ 44‴ stated in the *katapayadi* notation).

Multiply repeatedly the unit measurement, (which is the radius), by the square of the arc and divide by the square of the even numbers (2, 4 etc) decreased by that number and then multiplied by the square of the radius; the first is, however, to be divided by twice the radius. Place the results one below the other and subtract each from the one above it. To derive the sara-s (tabular versines of the arc) which are collected (and stated, in order in the mnemonic verse) beginning with the expression *stena* (i.e., 0° 0″ 6‴).'[1]

5. Infinite G. P. Convergent series

The credit for enunciating in India, for the first time, a formula for the sum of an infinite convergent geometrical progression, goes to Nilakantha Somayaji (born 1444). This he gives in his *Aryabhatiya-Bhasya*, while explaining the process of derving the arc of a circle in terms of the chord by means of a computation which involves the summing up of an infinite covergent G P series: *evam yas tulyaccheda-paramabhaga-paramaparaya anantaya api samyogah, tasya anantanam api kalpyamanasya yogasya adyavayavinah parasparamsacchedad ekonacchedamsasadhyam sarvatrapi samanam eva* (*Bhasya on Aryubatiya*, Ganita 17, end., Trivandrum, 1930, p. 106). 'Thus the sum of an infinite series, whose later terms (after the first) are got by diminishing the preceding one by the same divisor, is always equal to the first term divided by one less than the common mutual divisor '

In continuation of this enunciation, Nilakantha elaborately demonstrates it for a finite G P. and also for an infinite decreasing G. P.

6. Lhuiler's formula for the Circum-radius of a Cyclic quadrilateral

In Western mathematics, the eighteenth century mathematician Luhiler is credited with the discovery, in 1782, of an expression for the circum-radius of cyclic quadrilateral. In india, we find the same formula enunciated by the Kerala astronomer Paramesvara (c. 1360-1455) in his commentary on *Lilavati*, in the following lines:

doṣṇām dvayor dvayor ghātayutānām tisṛṇām vadhāt I
ekaikonet arattraikyam catuṣkavdhabhājitam II
labdhamūlena yadvṛttam viṣkambhārdhena nirmitam I
sarvam caturbhujakṣetram tasminn evāvatiṣṭhate II

'The three sums of the products of the sides, taken two at a time, are to be multiplied together and divided by the product of the sums of the sides taken three at a time and diminished by the fourth. If a circle is drawn with the square root of this quantity as radius, the

whole quadrilateral will be situated inside it.'

Thus, if a, b, c, d are the sides and r the circum-radius,

$$r = \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(a+b+c-d)(b+c+d-a)(c+d+a-b)(d+a+b-c)}}$$

The rationale of this formula has also been given in the 16th cent. Kerala commentary *Kriyakramakari on the Lilavati*

7. Gregory and Leibnitz's Series for the Inverse tangent

The power series for arc tan x which was enunciated for the first time in the United Kingdom in 1671 by the Scottish mathematician James Gregory (1638-75) and in Europe in 1673 by the German mathematician and philosopher Gottfried Wilhelm Leibnitz (1646-1716), in the case of x-1, may be stated as follows: In the case of infinite series of powers of x representing an arc of a circle of unit radius which subtends at the centre of the circle an angle whose tangent (x) does not exceed unity,

arc tan $x = x - x^3/3 + x^5/5 - \ldots\ldots (|x| \leq 1)$

In India, this series was enunciated by Madhava of Sangamagrama (1350-1410), nearly three centuries before it was discovered in the West. Madhava's enunciation of the formula is contained in the following lines:

iṣṭajyā-trijyayōr ghātāt kotyāptam prathamam phalam I
jyāvargam guṇakam kṛtvā koṭivargam ca harakam II
prathamādiphalebhyo tha neyā phalakṛtir muhuh I
eka-tryādy-ojasankhyābhir bhaktesv eteṣv anukramāt II
ojanām samyutes tyaktvā yugmayogam dhanur bhavet I
doḥ-kotyor alpam eveha kalpanīyam iha smṛtam I
labdhīnam avasānam syān na tathāpi muhuḥ kṛte II

'Obtain first the result of multiplying the jya (of a given dhanus) by the trijya and dividing the product by the koti (of the dhanus). Multiply this result by the square of the jya and divide the square by

the koti. Thus we obtain a second result,(as also) a sequence of the further results by repeatedly multiplying by the square of the jya and dividing by the odd numbers 1,3,5,etc , after this, add all the odd terms and substract from them all the even terms (without disturbing the order of the terms). Thus is obtained the dhanus whose two elements should be taken as the jya, since, otherwise, the series obtained will be non-finite (in value)'.

According to the above formula, if R is the radius and c are the sine and cosine chords of the arc,

the arc = $sR / c - sR / 3c \times s^2/c^2 + sR / 5c\ s^4/c^4$-.

which is Gregory's general series for arc tan x.

8. Leibnitz's Power Series for π

vyāse vāridhi-nihate rūpaṛte vyāsagarabhiate/ tri-śarādhi-
viṣamasankhyā-bhaktam ṛṇam svam pṛthak kramāt kuryat//

, Multiply the diameter by 4. Subtract from it and add to it alternately the quotients obtained by dividing four times the diameter to the odd integers 3, 5 etc.'

For a circumference C of a circle of diameter D, this gives the formula: C, (i,e.,π D) = 4D - 4D/3 + 4D/5......or π /4 = 1 = 1/3-1/5.....[1]

9. Aproximation to the value of π

Continuing his enunciation of the circumference of a circle, as given above, Madhava goes on to give a rational approximation to its value and, through it, the value of π:

Yatsaṅkhyayā' tra haraṇe Kṛte nivṛttā hṛtis tu jāmitayā I
tasyā urdhvagatayās samasaṅkhyā taddalam guṇo'nte syat II
tadvargai rūpahato hāro vyasābdhighatah prāgvat I
tasyām āptam svamṛṇe kṛte dhane śodhanañ ca karaṇīyam I
sukṣmah paridhiḥ sa syāt bahukṛtvo haraṇto 'tisuksma's ca II

' Let the process stop at a certain stage, giving rise to a 'finite sum'. Multiply four times the diameter by half the even integer subsequent to the last odd integer increased by unity The result is

the correction to be added to or subtracted from our 'finite sum', the choice of addition or subtraction depending on the sign of the last term in the sum. The final result is the circunferences determined more accuarately than(that obtained) by taking a large number oif terms,i.e.,terms going beyond the stage at which we stopped.'

The formula enunciatede here, which gives the value of π to an advanced degree of accuracy, may be expressed thus.

$$C = 4D\{1 - \frac{1}{3} + \frac{1}{5} - \ldots \pm \frac{1}{n} \pm \frac{(n+1)/2}{(n+1)^2+1}$$ where n is large.

$$\therefore \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \ldots \pm \frac{1}{n} \pm \frac{(n+1)/2}{(n+1)^2+1}$$

Better correction is suggested by Madhava in the following lines:

asmāt sūksmataro 'nyo vilikhyate kaścanāpi samskārah I
ante samasaṅkhadalavargas saiko guṇaeḥ, sa eva punoḥ II
yugaguṇito rūpayutaḥ samasankhyādalahato bhaved hāraḥ I
trisārādiviṣmasankhyāharaṇat param etad eva vā kāryam II'

'A correction still more precise is being stated here The multiplier is the square of half the even integer (next greater than then last odd-interger divisor) increased by unity, and then multiplier multiplied by 4, then increased by unity, and then multiplied by half the even integer (already defined), is the divisor This correction may be applied after the division by the odd integers 3,5 etc (indicated in the previous method and the calculation followed).'

This formula which gives a still more accurate value of π may be expressd thus.

$$C = 4D\{1 - \frac{1}{3} + \frac{1}{5} - \ldots \pm \frac{1}{n} \pm \frac{(n+1/2)^2+1}{[(n+1/2)^2.4+a](n+1/2)}\}$$

$$\pi / 4 = 1 - \frac{1}{3} + \frac{1}{5} - \ldots \pm \frac{1}{n} \pm \frac{(n+1/2)_2+1}{[(n+1/2)^2\,4+1]\,(n+1/2)}$$ where n is odd and large.

Elsewhere, Madhava gives a series for π , different. from that of Leibnitz. Thus, in continuation of the formula for the inverse tangent given in the lines istajyatrijyayor gatat etc., he mentions the measure of the circumference when the arc calculated is half of circumference, thus:

vyasavargād ravihatāt padam syāt prathamam phalam I
tadāditas trisaṅkhyāptam phalam syād uttarottaram II
rūpādyayugmasaṅkhyābhir hṛteṣv eṣu yathākramam I
viṣamāṇam yutes tyaktavā samam hi paridhir bhavet II

'Multiply the square of the diameter by 12 and extract the square root of the product. That is the first term. Divide the terms, in order, by the odd numbers (1,3,5) etc. Add the odd-order terms to and subtract the even-order terms from the preceding. The result will give the circumference:'

The above enuciation gives the following formula for the circumference:

$$C \text{ (i.e, } \pi D) = \sqrt{12D^2} - \frac{\sqrt{12D^2}}{3.3} + \frac{\sqrt{12D^2}}{5.3^2} - \frac{\sqrt{12D^2}}{7.3^3}$$

$$= \sqrt{12}D\{1 - \tfrac{1}{3.3} + \tfrac{1}{5.3^2} - \tfrac{1}{7.3^3} \ldots \therefore \pi = \sqrt{12}\{1 - \tfrac{1}{3.3} + \tfrac{1}{5.3^2} - \tfrac{1}{7.3^3} \ldots$$

It would be interesting to derive from Madhava's above-noted value of π ,the following relation between circumference and diameter given by him:

vibudha-netra-gaj-āhi-hutāśana- tri-guṇa-veda-bha-vāraṇa-bāhavāḥ I
nava-nikharva-mite vṛitivistare paridhi-mānam idam jagadur budhāḥ II

That is, for a diameter of 9^{11}, the circumference is 28, 27, 43, 88, 233. The value of π from this, correct to eleven decimal places, would be 3.13159265359, which accords closely with the modern value of π , which is 3.14159265.

Using the above relationship of the diameter and the circumference, Madhava has derived the 24 mahajyas (accurate sign-

chords) which he has depicted in the mnemonic beginning with srestham nama varisthanam (0°-224′ - 50″-22‴)[1]. It has also been verified that the value of the radian assumed by Madhava in this evaluation corresponds to 3437′-44″-48‴-22⁗. which is remarkably close to its modern approximation, *viz*, 3437′.74677078=3437′-44′-48′ - 22″ ′,49.

Quotations and Description by Dr. K. V. Sarma, A Histroy of the Kerala School of Hindu Astronomy, Visweswaranand Institute, Hoshiarpur, 1972.

Period of the books / authors refered in the text:

Aryabhateeyam of Aryabhatta I	499 AD
Siddanta Darpana of Neelakanta	1465 AD
Panchasiddanteka of Varahamihira	587 AD
Sishyadhi Vruddhi Tantra of Lallacharya	749 AD
Laghumanasa of Manjulacharya	932 AD
Laghubhaskareeyam commentary by Sankaranarayana	869 AD
Sishyadhi Vruddhi Tantra Commentary by Mallikarjuna Suri	1178 AD
Laghubhankareeyam of Bhaskarcharya I	629 AD
Suryasiddhanta - Anonymous	600 AD
Brahma sphuta siddhanta of Brahmagupta	628 AD
Bruhat Samhita of Varahamihira	570 AD
Bhadrabahu Samhita	400 AD
Bhaskaracharya's (I) Commentry for Aryabhateeya	629 AD
Ganita Sara Sangraha of Mahaveeracharya	750 AD
Vyasa Bhashya Yoga Sutra	600 AD
Siddhanta Sekhara of Sripathi	1039 AD
Leelavati of Bhaskaracharya II	1114 AD
Beejaganita of Bhaskaracharya II	1114 AD
Vishnu smruthi - Anonymous	200 BC
Kriyakramakari of Madhavacharya	1340 AD
Karanapaddnati of Puthumana Somayaji	1430 AD
Mahabhaskareeya of Bhaskaracharya I	629 AD
Khandakhadyaka of Brahmagupta	665 AD
Tantra Sangraha of Neelakanta	1465 AD
Davala Teeka of Virasena	816 AD

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